STUDY OF HYDRAULIC RESISTANCES IN THE PISTON FLOW OF MIXTURES OF AIR WITH LIQUIDS OF DIFFERENT VISCOSITIES

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A semiempirical method is proposed for calculating the hydraulic resistance of a two-phase piston flow.

The engineering methods used to calculate the piston flow of a gas–liquid mixture are based on empirical relations with different structures, these relations describing the dependence of integral quantities on physical and flow-rate parameters of the flow [1-4]. This situation prevails because, until now, the main method of studying two-phase flows has been the empirical method.

The substantial volume of data which has now been accumulated on the characteristics of piston flows makes it possible to perform engineering calculations with good accuracy in some cases. However, in those cases where the actual physical properties of the components of the mixture differ appreciably from those in the experiment, the discrepancy between the theoretical data and empirical results becomes so great as to render the models used unfit for practical calculations. It then becomes necessary to conduct further studies of piston flow within a broad range of variation in the physical properties of the mixture components, particularly for laminar flow of the liquid phase. There are no such results in the literature except for [1], where the data are presented in such a form that their practical use is almost impossible.

In the present work, experimental studies were conducted with the movement of a mixture of air with liquids of different viscosity in a horizontal glass tube with an inside diameter d = 13 mm and a length l = 6.7 m. The liquid component of the mixture was alternately water $\mu_1 = 1 \cdot 10^{-3}$ N·sec/m^2, diesel fuel $\mu_1 = 3.6 \cdot 10^{-3}$ N·sec/m^2, "Veretennoe AU"-grade oil $\mu_1 = 41.9 \cdot 10^{-3}$ N·sec/m^2, and two concentrations of solutions of this oil in diesel fuel $\mu_1 = 21.1 \cdot 10^{-3}$ and $\mu_1 = 10 \cdot 10^{-3}$ N·sec/m^2. Several series of tests were conducted for each viscosity of the liquid phase. With a fixed value of the Froude number of the mixture $Fr_c = U_c^2/gd$ in each test, the volumetric discharge gas content β_2 was varied from 0 to 1.

The actual volumetric phase concentrations φ_1 and φ_2 were measured by the method of cutting the testing unit off from the main. The distance between the cutoffs was 4.16 m. The first pressure sampling in the flow direction was located 3.07 m from the mixer. It should be noted that the results of these experiments agreed well with the empirical relation in [3].

The pressure drop was measured with a U-shaped water-column gauge. Separating vessels were used to keep water out of the impulse lines for sampling pressure. The distance between the pressure samplings was 1.355 m. The results obtained were analyzed by different methods and compared with calculated values. For the comparison, we chose the methods which have been most widely used in engineering practice: that of Martinelli [1], the standard method of the I. I. Polzunov Central Scientific Research, Planning, and Design Boiler and Turbine Institute (TsKTI) [2], the method in [3], and the method of Armand [4].

The results of calculations in [1, 3, 4] for the movement of an air–water mixture and a mixture of air and diesel fuel are in satisfactory agreement with the experimental data. In our opinion, this is due to the fact that the flow of the liquid phase at a rate equal to the flow rate of the mixture is turbulent in these cases, and the empirical relations in [2-4] were obtained specifically for this case. As regards the method of [1], this study examined three possible variants: turbulent flow of the gas and liquid, laminar flow of the liquid and turbulent flow of the gas, and laminar flow of the liquid and gas.

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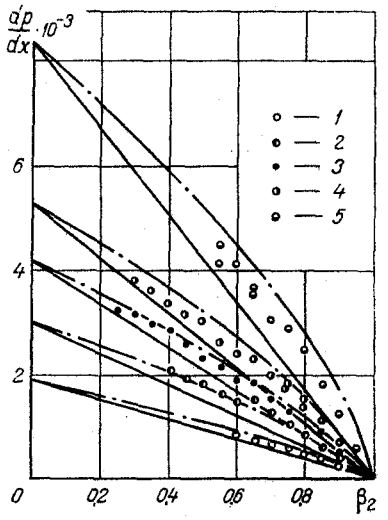

Fig. 1. Comparison of experimental data with a liquid-phase viscosity $\mu_1 = 41.9 \cdot 10^{-3}$ N·sec/m^2 against results calculated by Martinelli's method [1] (dot–dash curves) and the method in [3] (solid curves): 1) $Fr_c = 0.4$; 2) 1; 3) 2; 4) 4; 5) 8. dP/dx, Pa/m.

Figure 1 compares experimental and theoretical values of the pressure gradient calculated by the methods in [1, 3]. They agree satisfactorily with the results calculated by Martinelli's method (laminar–laminar flow).

The method in [3] is based on the assumption of a linear decrease in the pressure gradient with an increase in the discharge gas content, an assumption which is quite justified for turbulent flow of the liquid phase. In laminar flow, this linear relation no longer holds, and the empirical points are located above the theoretical curve. The causes of the deviation of the experimental data from the theoretical results obtained by the method in [3] should be looked for in the principle of analyzing the test data in the form of corrected resistance coefficients:

$$\psi = \lambda_c / \lambda_0, \tag{1}$$

where λ_0 is determined by the Reynolds number of the mixture $Re_c = U_c d/\nu_c$ ($1/\nu_c = \beta_1/\nu_1 + \beta_2/\nu_2$) and the relative roughness k_e/d. The coefficient of hydraulic resistance of the mixture λ_c is determined through the actual dynamic head:

$$-\frac{dp}{dz} = \lambda_c \frac{1}{2d}\left(\frac{\beta_1^2}{\varphi_1}\rho_1 + \frac{\beta_2^2}{\varphi_2}\rho_2\right) U_c^2. \tag{2}$$

To calculate ψ, it is suggested that an empirical formula having the following form for the case $\rho_1 >> \rho_2$ be used

$$\psi = \frac{\lambda_1}{\lambda_0}\frac{\varphi_1}{\beta_1}, \tag{3}$$

where λ_1 is calculated from the Reynolds criterion $Re_1 = U_c d/\nu_1$ and the roughness.

Such an analysis presumes that the effect of the viscosity of the liquid phase on λ_c is completely accounted for by the value of λ_0 (through the value of Re_c). This assumption is valid in the region of high Reynolds numbers, when λ_0 depends mainly on the roughness and remains nearly constant for the given tube throughout the region of the test parameters. At low to moderate values of Re_c, the value of λ_0 usually changes appreciably. This cannot be reflected in a change in λ_c. Also, the very determination of Re_c with respect to "additive viscosity" is quite tentative and does not have a clear physical meaning.

In accordance with the standard method of the All-Union Institute of Heat Engineering (VTI) and the TsKTI, the pressure loss due to friction is calculated for a two-phase flow from the formula

$$\Delta p_c / \Delta p_0 = 1 + C x_2 (\rho_1/\rho_2 - 1), \tag{4}$$

where Δp_0 is the pressure loss to friction in the movement of a liquid with the same mass rate; x_2 is the mass discharge gas content; C is a coefficient determined from nomograms. For the case of atmospheric pressure, C = 1.5.

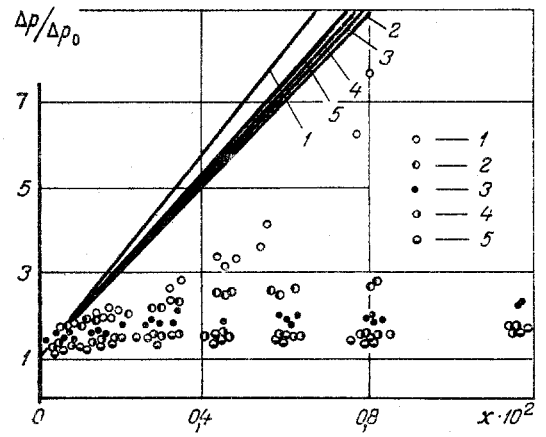

Fig. 2

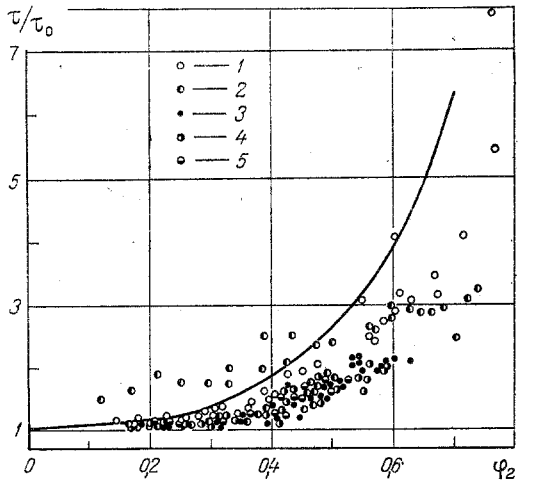

Fig. 3

Fig. 2. Comparison of test data with results calculated by the TsKTI method [2] (solid lines): 1) $\mu_1 = 1 \cdot 10^{-3}$ N•sec/m²; 2) $3.6 \cdot 10^{-3}$; 3) $10 \cdot 10^{-3}$; 4) $21.1 \cdot 10^{-3}$; 5) $41.1 \cdot 10^{-3}$.

Fig. 3. Fig. 3. Comparison of experimental data with results calculated by the method of Armand [4] (solid curve): 1) $\mu_1 = 1 \cdot 10^{-3}$ N•sec/m²; 2) $3.6 \cdot 10^{-3}$; 3) $10 \cdot 10^{-3}$; 4) $21.1 \cdot 10^{-3}$; 5) $41.9 \cdot 10^{-3}$.

The analysis of our data by the method in [2] is presented in Fig. 2. The layering of the test points for viscosity is noticeable. An increase in μ_1 leads to a decrease in the ratio $\Delta p_c/\Delta p_0$. The same figure shows results of calculations with Eq. (4). It is evident that the test and calculated data diverge a great deal, especially for similar liquid viscosities.

Figure 3 shows the results of analysis of our data by the method of Armand [4] in the coordinates τ_c/τ_0, φ_2. The theoretical curve (solid line) corresponds to the empirical formula

$$\tau_c / \tau_0 = 1 / \varphi_1^{1,42}, \tag{5}$$

where τ_0 is the friction on the tube wall in the movement of a uniform liquid at a velocity equal to its corrected velocity in the two-phase flow.

The discrepancy between the test and calculated data is explained by the effect of the viscosity of the liquid phase on the true volumetric liquid content φ_1. At the same time, as in all of Armand's tests — which were the basis for the derivation of (5) — we took

$$\varphi_2 = 0.833\beta_2, \tag{6}$$

which corresponds to the flow of an air–water mixture.

Thus, the methods of calculating the resistance to the piston flow of a two-phase mixture used here are limited to the range of physical properties of the mixture components in which the experiments on which the methods are based were conducted. Some of them, however, make it possible to perform calculations with satisfactory accuracy [1, 3].

The most universal and substantiated method of calculating a two-phase flow can be created by means of the semiempirical method of investigation. The changeover from empirical methods to semiempirical theories in the study of uniform liquids was a new stage in the development of hydrodynamics. However, the theoretical formulas obtained by the semiempirical approach do not improve the accuracy of the calculations in most cases, but they are more physically substantiated, contain a minimum number of empirical coefficients, and are the most universal, i.e., they can be used in a broader range of flow parameters.

In recent years there have appeared works in which the aim was to develop a semiempirical theory for two-phase flows [5, 6]; a brief look is taken at their characteristics in [7].

The study [7] proposed the hypothesis of "large-scale mixing length" for a piston gas–liquid flow. It examined the case when the motion of the carrying phase (the liquid) is turbulent. The components of the shear-stress tensor are represented in the form of the sum of two terms — small-scale and large-scale. The small-scale component is calculated by means of familiar semiempirical theories on the turbulence of a uniform liquid, while an additional hypothesis was proposed to calculate the large-scale component. The essence of this hypothesis is that additional "large-scale" mixing occurs with the movement of gas occlusions (plugs) relative to the surrounding liquid. This additional mixing results in large-amplitude, low-frequency pulsations of the hydrodynamic quantities, which in turn leads to the appearance of an additional term of the form $\overline{u_g' v_g'}$ in the expression for the shear stresses and, thus, to an increase in the coefficient of hydraulic resistance of the two-phase mixture.

We will attempt to use the method developed in [7] to analyze piston two-phase flow in the case of laminar flow of the carrying phase.

Assuming that the gas occlusions move as undeformable cavities and that there are no shear stresses in them, we write the equation for the shear stress:

$$\tau = \alpha_1 (\tau_m + \tau_g), \tag{7}$$

where α_1 is the probability of existence of liquid at a given point (local concentration). The value of τ_m will be calculated using the familiar law of viscous flow

$$\tau_m = \mu_1 \frac{du_1}{dy}. \tag{8}$$

We represent the large-scale component of shear stress in a manner similar to [7]:

$$\tau_g = \alpha_2 \mu_g \frac{d(u_1 - u_2)}{dy}, \tag{9}$$

where μ_g is the "large-scale viscosity," due to the additional mixing that occurs with the passage of the gas occlusion.

The probability of existence of the gas phase α_2 enters into Eq. (9) because the large-scale stresses appear only with the passage of the gas phase through the flow section being examined [7].

Since we are examining gaseous occlusions in the form of undeformable cavities, the velocity of the gas phase across the flow does not change and is equal to its own mean value $u_2 = U_2$. With allowance for this, Eq. (7) takes the form

$$\tau = \alpha_1 \frac{U_1}{U_c} (\mu_1 + \alpha_2 \mu_g) d \left[u_c \left(\frac{u_1}{U_1} \Big/ \frac{u_c}{U_c} \right) \right] \Big/ dy. \tag{10}$$

If we use the usual premise [5] regarding the similitude of the velocity profile of the liquid phase and the local-concentration profile of the gas phase $u_1/U_1 = \alpha_2/\varphi_2$, as well as the familiar relations $U_1 = \frac{\beta_1}{\varphi_1} U_c$, $U_2 = \frac{\beta_2}{\varphi_2} U_c$, it is easily shown that the dimensionless velocity profiles of the liquid phase and the mixture u_1/U_1 and u_c/U_c are similar:

$$\frac{u_c}{U_c} = \frac{\alpha_1 u_1 + \alpha_2 u_2}{U_c} = \left(\beta_2 + \beta_1 \frac{\alpha_1}{\varphi_1} \right) \frac{u_1}{U_1}. \tag{11}$$

It is not hard to see that the factor in the parentheses differs little from unity, since, with piston flow of the mixture, the local concentration of the liquid changes little across the flow. Thus, $u_1/U_1 \approx u_c/U_c$, and Eq. (10) takes the form

$$\tau = \alpha_1 \frac{\beta_1}{\varphi_1} (\mu_1 + \alpha_2 \mu_g) \frac{du_c}{dy}. \tag{12}$$

Following [7] and replacing the probability of existence of the phases α_1 and α_2 by the mean values across the flow φ_1 and φ_2, we finally obtain

$$\tau = \beta_1 (\mu_1 + \varphi_2 \mu_g) \frac{du_c}{dy}. \tag{13}$$

Determining the coefficient of hydraulic resistance of the mixture through the actual dynamic head (2), we integrate (13) and by the usual method [8] obtain the formula for calculating λ_c:

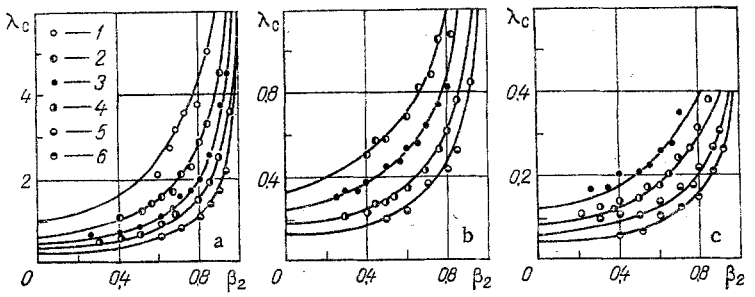

Fig. 4. Comparison of experimental data with semiempirical relation (14) (solid curves): 1) Fr_c = 0.4; 2) 1; 3) 2; 4) 4; 5) 8; 6) 16 (a — μ_1 = $41.9 \cdot 10^{-3}$ N·sec/m^2, b — $21.1 \cdot 10^{-3}$, c — $10 \cdot 10^{-3}$).

$$\lambda_c = \frac{64}{Re_1} \frac{\varphi_1}{\beta_1} \left(1 + \varphi_2 \frac{\mu_g}{\mu_1} \right). \tag{14}$$

Comparison of this expression with our test data showed good agreement when $\mu_g/\mu_1 = 1$ for all liquid viscosities. As an example, Fig. 4 shows results of such a comparison for three values of liquid-phase viscosity. The values of φ_1 and φ_2 needed for calculation with Eq. (14) were taken from our measurements.

It is very interesting to compare derived relation (14) with the method of calculating two-phase flows based on the concept of the effective viscosity of a mixture. Einstein's formula for the effective viscosity of a mixture with gas bubbles takes the form [9]:

$$\mu_{ef} = \mu_1 (1 + \varphi_2). \tag{15}$$

Both methods give the same results when the concentration of the gas phase is low and the difference between the volumetric and actual phase concentrations is not great. With an increase in gas content, the hydraulic resistance calculated from Eq. (14) increases considerably more rapidly than the value obtained from Eq. (15). The rapid increase in effective viscosity compared to the values calculated with Eq. (15) was observed experimentally in the case of the motion of suspensions [9].

Thus, the proposed semiempirical method has made it possible to obtain a theoretical formula which contains only one unknown μ_g/μ_1, and this unknown proved to be constant throughout the entire range of mixture velocities and liquid-phase viscosities.

NOTATION

U, mean velocity across the flow, m/sec; u, longitudinal component of velocity, m/sec; p, pressure, Pa; v, transverse component of velocity, m/sec; τ, shear stress, N/m^2; ρ, density, kg/m^3; μ, absolute viscosity, N·sec/m^2; ν, kinematic viscosity, m^2/sec; l, length, m; d, inside diameter of tube, m; k_e, equivalent roughness, m; z, coordinate (on tube axis), m; y, coordinate (distance from tube wall), m; β, volumetric discharge concentration of the phase, dimensionless; φ, true volumetric phase concentration, dimensionless; x, mass discharge phase concentration, dimensionless; λ, coefficient of hydraulic resistance, dimensionless; ψ, corrected coefficient of hydraulic resistance, dimensionless; C, coefficient in Eq. (4). Indices: 1, liquid; 2, gas; c, mixture; o, single-phase; m, laminar; g, large-scale. A bar denotes an average over time; ' denotes the fluctuation component. Criteria: Fr, Froude; Re, Reynolds.

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HEATING OF A MICROPOLAR LIQUID DUE TO VISCOUS ENERGY DISSIPATION IN CHANNELS. 1. POISEUILLE FLOW

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An analytical study is made of the effect of the internal microstructure of a liquid on its heating due to viscous energy dissipation.

In its different forms, the theory of micropolar liquids (MPL) [1] is currently used for theoretically describing transfer processes in liquids with an internal microstructure: liquid crystals, magnetic liquids, certain suspensions, and associated liquids. However, a comparatively large number of transfer coefficients (material constants) which until recently had no method of being determined are included in this theory. The studies [2, 3] proposed methods of determining different parameters characterizing the internal microstructure of a liquid. For example, the method of determining the material constants of a liquid in [3] is based on measurement of heating of the liquid as a result of viscous energy dissipation during Poiseuille flow in a plane channel in the case of constant channel-wall temperature.

The present work analytically solves a problem of the heating of an MPL flowing as a result of a fixed pressure gradient in the plane channel. We set thermal boundary conditions more general than those in [3] and take into account the change in temperature through the thickness of the channel walls. A numerical analysis is made of the dependence of the temperature field in the liquid on quantities characterizing its micropolarity. The magnitude of the dissipative heating of a liquid flowing in microcapillaries ($h \leq 10^{-5}$ m) is very small in the overwhelming majority of cases. However, we also have the goal of studying dissipative heating of the liquid under conditions where its value is sufficiently large for experimental determination (for example, with a pressure drop Δp = 20-40 atm).

In the second article we will solve a similar problem for Couette flow with a prescribed constant relative velocity of the channel walls. The role of microrotations of particles of the medium in the case of significant dissipative heating of the microstructural liquid is established for a broad range of practical instances, such as when two surfaces with an intervening liquid are moving at a comparatively high velocity relative to each other.

Let us examine the stabilized flow of an incompressible micropolar liquid under the influence of a constant pressure gradient dp/dx between parallel plates located a distance 2h from one another. The x axis of the Cartesian coordinate system coincides with the central line of the channel, while the y axis is perpendicular to the plates. In this case, the velocity vector $\vec{v}$ and the microrotation vector $\vec{\nu}$ only have nontrivial components $v_x(y)$ and $\nu_z(y)$, respectively. We will assume that the physical properties of the MPL are constant, i.e., we will ignore the effect of dissipative heating on them, as well as the body forces and their moments. Given these assumptions, the equations describing the flow of the MPL have the form

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